Theorem/Results Reference Sheet

Please remove this sheet before submitting your final exam.

Divergence Test

If $\lim_{n\to\infty} a_n \neq 0$,

then $\sum_{n=n_0}^{\infty} a_n$ diverges.

Geometric Series Test

$\sum_{n=n_0}^{\infty} ar^n$ converges if and only if $|r| < 1$.

If $|r| < 1$, $\sum_{n=n_0}^{\infty} ar^n = \dfrac{ar^{n_0}}{1-r}$

p-Series Test

$\sum_{n=1}^{\infty} \dfrac{1}{n^p}$ converges

if and only if $p > 1$

Integral Test

If $f(x)$ is positive, continuous, decreasing on $[n_0, \infty)$, then

the series $\sum_{n=n_0}^{\infty} f(n)$ converges if and only if the improper integral $\int_{n_0}^{\infty} f(x)\,dx$ converges.

Comparison Test

Let a_n and b_n be such that $0 \leq a_n \leq b_n$ for all $n \geq n_0$.

If $\sum a_n$ is divergent, then $\sum b_n$ is divergent.

If $\sum b_n$ is convergent, then $\sum a_n$ is convergent.

Limit Comparison Test

Let $a_n, b_n \geq 0$ for all n and let $c = \lim_{n\to\infty} \dfrac{a_n}{b_n}$.

If $0 < c < \infty$, then $\sum a_n$ and $\sum b_n$

both converge or both diverge.

Alternating Series Test and Alternating Series Estimation Theorem

If (1) $b_n > 0$ for all $n \geq n_0$, and (2) $b_{n+1} \leq b_n$ for all $n \geq n_0$ (i.e. b_n is decreasing), and (3) $\lim_{n\to\infty} b_n = 0$,

then $S = \sum_{n=n_0}^{\infty} (-1)^{n-1} b_n$ converges and for all $N \geq n_0$, $|\, S - S_N \,| = \left| S - \sum_{n=n_0}^{N} (-1)^{n-1} b_n \right| < b_{N+1}$.

Ratio Test

Let $S = \sum a_n$ be some series and let $L = \lim_{n\to\infty} \left| \dfrac{a_{n+1}}{a_n} \right|$

(i) If $L < 1$, the series is absolutely convergent.
(ii) If $L = 1$, the Ratio Test is inconclusive.
(iii) If $L > 1$ or $L = \infty$, the series is divergent.

Root Test

Let $S = \sum a_n$ be some series and let $L = \lim_{n\to\infty} \sqrt[n]{|a_n|}$

(i) If $L < 1$, the series is absolutely convergent.
(ii) If $L = 1$, the Root Test is inconclusive.
(iii) If $L > 1$ or $L = \infty$, the series is divergent.

Important Power Series Representations

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \text{ on } (-1,1) \qquad \sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \text{ on } \mathbb{R}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \text{ on } \mathbb{R} \qquad \cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \text{ on } \mathbb{R}$$

Taylor's Inequality

Let $T_n(x) = \sum_{n=0}^{\infty} \dfrac{f^{(n)}(a)}{n!}(x-a)^n$. Then,

$$|R_n(x)| = |f(x) - T_n(x)| < \frac{|x-a|^{n+1}}{(n+1)!}(M)$$

for any $M > 0$ such that for all $n \geq 0$,

$f^{(n+1)}(t) \leq M$ for all $t \in [0, x]$.

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